الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيــة هندسة الحاسوب والمعلوماتية
Computer and Informatics Engineering Faculty

Electric Circuits I

Dr. Eng.
Hassan M. Ahmad

Hassan.Ahmad@spu.edu.sy, istamo48@mail.ru

Chapter 6
Capacitors and Inductors

6.1 Capacitors

6.2 Series and Parallel Capacitors

6.3 Inductors

6.4 Series and Parallel Inductors

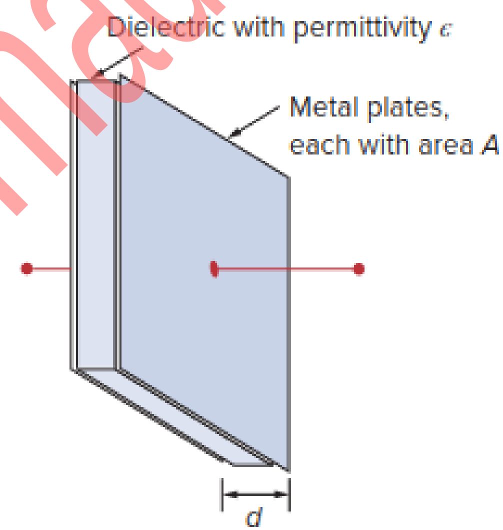
6.1 Capacitors

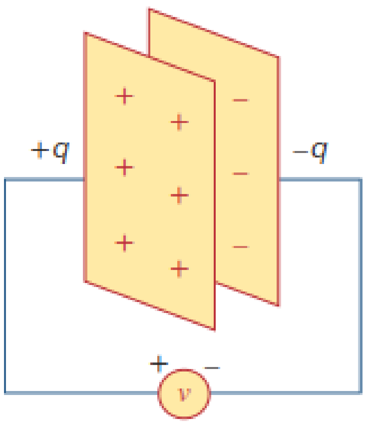
- A capacitor is a passive element designed to **store energy** in its **electric field.**
- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).
- The amount of **charge** stored, represented by q:

$$q = Cv$$

- **Capacitance** C is the **ratio** of the charge q on one plate of a capacitor to the voltage difference v between the two plates, measured in **farads (F)**.
- **For example**, for the parallel plate capacitor shown in Fig., the capacitance is given by

$$C = \frac{\varepsilon A}{d}$$

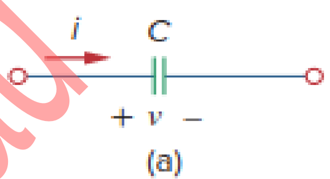


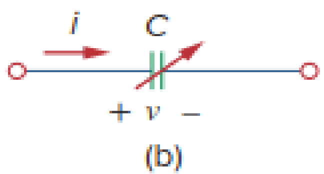


where ε is the permittivity (سماحية) of the dielectric material between the plates, A is the surface area (مساحة السطح) of each plate, d is the distance between the plates.

Types of Capacitors

- Typically, capacitors have values in the picofarad (pF,10^{-12}), nanofarad (nF,10^{-9}), microfarad (μF,10^{-6}).
- There are two types: (a) fixed and (b) variable, with circuit symbols as shown in Fig.
- If $v > 0$ and $i > 0$ or if $v < 0$ and $i < 0$, the capacitor is being charged, and if $v \cdot i < 0$, the capacitor is discharging.
- **Fixed capacitors**: (1) polyester, (2) ceramic, (3) electrolytic.
- **Variable capacitors**: (4) trimmer, (5) filmtrim.



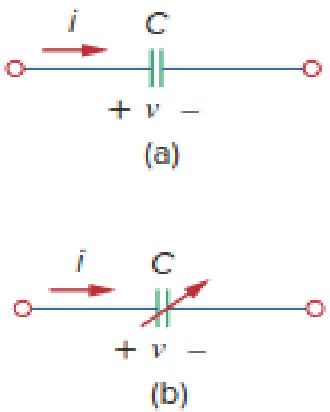


- In addition, capacitors are used to block dc, pass ac, shift phase, store energy, start motors, and suppress noise.

Current & Voltage of the capacitor

- If $v > 0$ and $i > 0$ or if $v < 0$ and $i < 0$, the capacitor is being charged, and if $v \cdot i < 0$, the capacitor is discharging.
- The current-voltage relationship of the capacitor is defined as following:

$$q = Cv \text{ and } i = \frac{dq}{dt} \Rightarrow \boxed{i = C\frac{dv}{dt}}$$



- The voltage-current relation of the capacitor can be obtained by $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$ or $\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)}$
- where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0.
-

Power & Energy of the capacitor

- The instantaneous power delivered to the capacitor is $p = vi = Cv\frac{dv}{dt}$
- The energy stored in the capacitor is

$$w = \int_{-\infty}^{t} p(\tau)d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} vdv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

- We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$.
- Thus, $\boxed{w = \frac{1}{2}Cv^2}$. We may rewrite energy stored as $w = \frac{q^2}{2C}$

- **Important properties of a capacitor:**

1) A capacitor is an open circuit to dc voltage (the current through the capacitor is zero). (**dc conditions**)

2) The voltage on a capacitor cannot change abruptly (فجأة).

Example 6.1

1) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
2) Find the energy stored in the capacitor.

Solution

1) Since $q = Cv$, ➔ $q = 3\times10^{-12}\times 20 = 60\,\text{pC}$

2) The energy stored is $w = \frac{1}{2}Cv^2 = \frac{1}{2}\times 3\times10^{-12}\times(20)^2 = 600\,\text{pJ}$

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Example 6.2

The voltage across a 5-μF capacitor is $v(t) = 10\cos 6000t$ V

Calculate the current through it.

Solution

$$i(t) = C\frac{dv}{dt} = 5\times10^{-6}\frac{d}{dt}(10\cos 6000t)$$

$$= -5\times10^{-6}\times 6000\times 10\sin 6000t$$

$$= -0.3\sin 6000t \text{ A}$$

Example 6.3

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution

$$v = \frac{1}{C}\int_0^t i\,d\tau + v(0) \quad \text{and} \quad v(0) = 0$$

$$v = \frac{1}{2\times10^{-6}}\int_0^t 6e^{-3000t}\times10^{-3}\,d\tau = \frac{3\times10^{3}}{-3000}e^{-3000t}\Big|_0^t = \left(1-e^{-3000t}\right) \text{ V}$$

Example 6.4

Obtain the energy stored in each capacitor in Fig. (a) under dc conditions.

Solution

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. (b).

The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division (CDR) as

$$i = \frac{3}{3+2+4} 6\,\text{mA} = 2\,\text{mA}$$

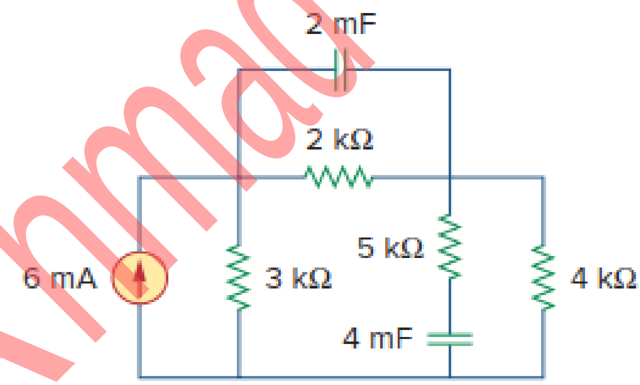
Hence, the voltages v_1 and v_1 across the capacitors are

$$v_1 = 2000i = 4\,\text{V}; \quad v_2 = 2000i = 8\,\text{V}$$

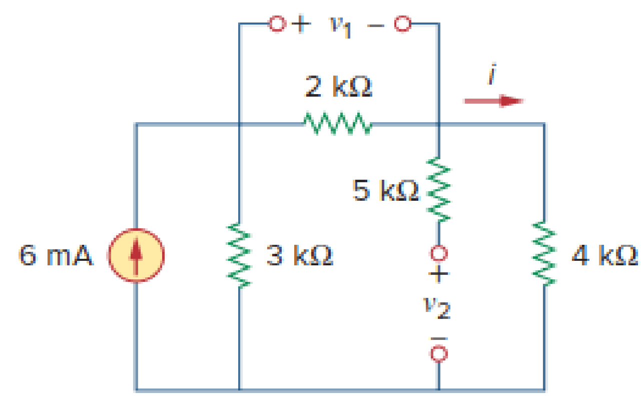
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}\times(2\times10^{-3})(4)^2 = 16\text{ mJ}$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}\times(4\times10^{-3})(8)^2 = 128\text{ mJ}$$



(a)



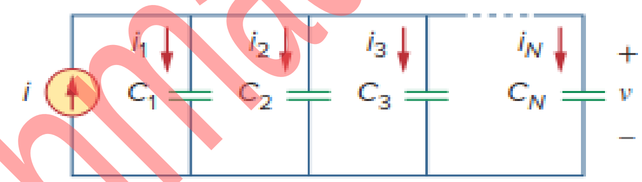
(b)

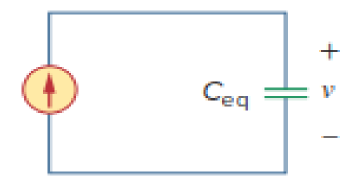
6.2 Series and Parallel Capacitors

Parallel capacitors

The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_N$$

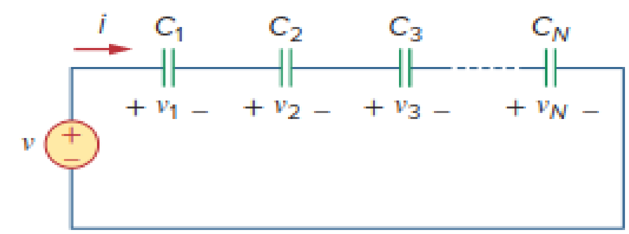




Series capacitors

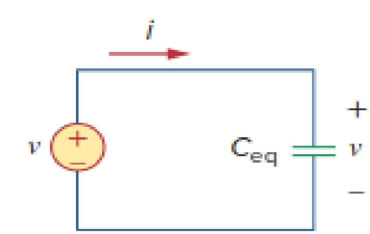
The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_N}$$



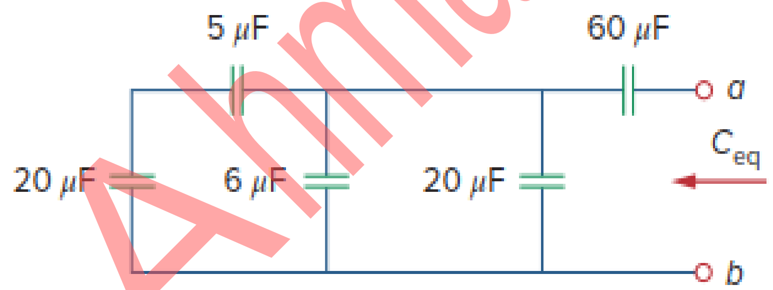
For $N = 2$ (i.e., two capacitors in series):

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



Example 6.5

Find the equivalent capacitance seen between terminals *a* and *b* of the circuit in Fig.



Solution

20- μF and 5- μF in series:

$$\frac{20\times 5}{20+5} = 4\,\mu\text{F}$$

4-μF in parallel with the 6- μF and 20- μF:

$$4+6+20 = 30\,\mu\text{F}$$

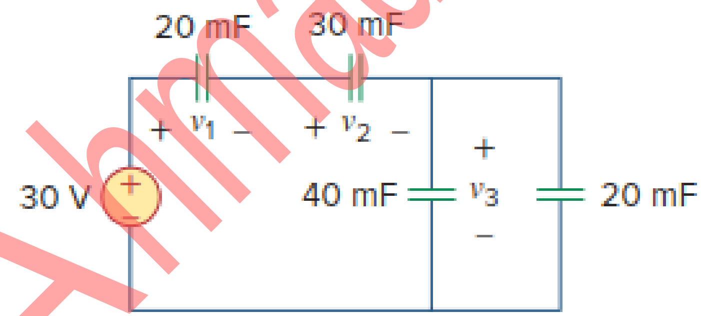
30-μF in series with 60- μF:

$$C_{eq} = \frac{30\times 60}{30+60} = 20\,\mu\text{F}$$

Example 6.6

For the circuit in Fig., find the voltage across each capacitor.

Solution



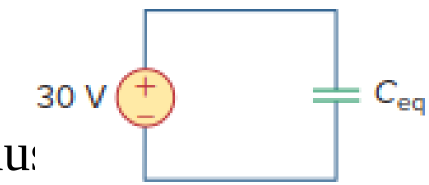
We first find the equivalent capacitance C_{eq}:

$$40\,\text{mF} \,||\, 20\,\text{mF} \Rightarrow 40 + 20 = 60\,\text{mF}$$

60-mF in series with 20-mF and 30-mF:

$$\frac{1}{C_{eq}} = \frac{1}{60} + \frac{1}{30} + \frac{1}{20}\,\text{mF} \Rightarrow C_{eq} = 10\,\text{mF}$$

The total charge is $q = C_{eq} v = 10 \times 10^{-3} \times 30 = 0.3\ \text{C}$



This is the charge on the 20-mF and 30-mF capacitors, becau s with the 30-V source. (the charge acts like current, since $i = dq/dt$.)

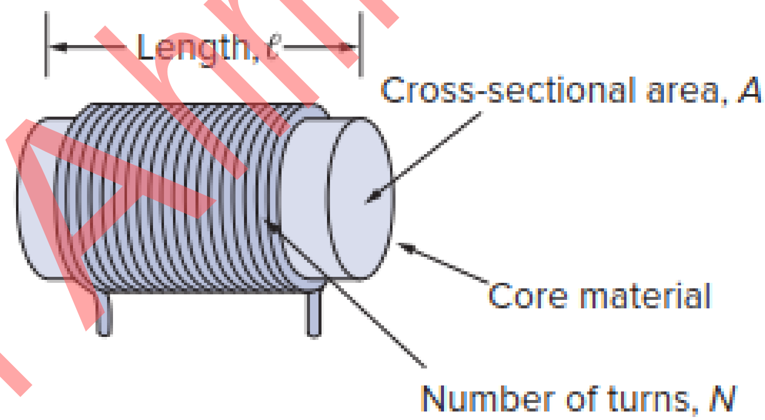
Therefore, $v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\,\text{V}; \quad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10\,\text{V}$

Using KVL: $v_3 = 30 - v_1 - v_2 = 5\ \text{V}$

Alternatively, the 20-mF, 30-mF and 60-mF in series, so they have the same charge on it. Hence, $v_3 = \frac{q}{60\,\text{mF}} = \frac{0.3}{60 \times 10^{-3}} = 5\,\text{V}$

6.3 Inductors

- An **inductor** is a passive element designed to store energy in its magnetic field.
- An inductor consists of a coil of conducting wire.



- The voltage across the inductor is

$$v = L\frac{di}{dt}$$

where L is the constant, called the inductance of the inductor.

- **Inductance** is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H):

$$L = \frac{N^2 \mu A}{l}$$

where:

N is the number of turns, ℓ is the length, A is the cross-sectional area, and μ is the permeability (النفاذية) of the core.

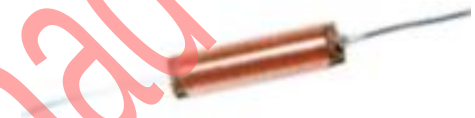
Common Types of Inductors

Various **types** of inductors:

a) solenoidal wound (ملفوف بشكل لولبي) inductor,

b) toroidal (حلقي) inductor,

c) axial lead (سلك موصل محوري) inductor.

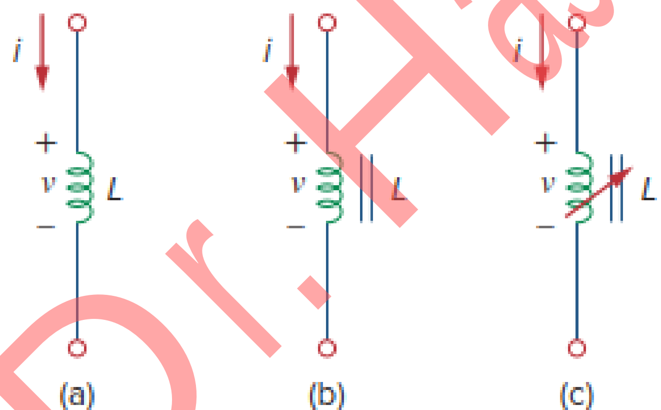


(a)

(b)

The circuit **symbols** for inductors:

(a) air-core, (b) iron-core, (c) variable iron-core.



Current-Voltage relationship , Power & Energy of Inductor

- The **current-voltage** relationship is obtained as: $$di = \frac{1}{L} v dt \Rightarrow i = \frac{1}{L}\int_{-\infty}^{t} v(\tau) d\tau$$

or $$i = \frac{1}{L}\int_{t_0}^{t} v(\tau) d\tau + i(t_0)$$

where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty) = 0$.

- The **power** delivered to the inductor is $$p = vi = (L\frac{di}{dt})i$$

- The **energy** stored: $$w = \int_{-\infty}^{t} p(\tau) d\tau = L\int_{-\infty}^{t} \frac{di}{d\tau} i d\tau = L\int_{-\infty}^{t} i\, di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

  Since $i(-\infty) = 0$, $$w = \frac{1}{2}Li^2$$

Important properties of an inductor:

1) An inductor acts like a short circuit to dc.
2) The current through an inductor cannot change instantaneously.

Example 6.7

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A .

Find the voltage across the inductor and the energy stored in it.

Solution

Since $v = L\, di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1-5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)\times 100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Example 6.8

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at t = 5 s. Assume $i(v) > 0$.

Solution

Since L = 5 H ➔ $$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0) = i = \frac{1}{5}\int_0^t 30\tau^2 d\tau + 0 = 6\times\frac{t^3}{3} = 2t^3 \text{ A}$$

The power : $$p = vi = 30t^2 \times 2t^3 = 60t^5 \text{ W}$$

The energy stored: $$w = \int p\,dt = \int_0^5 60t^5 dt = 60\left.\frac{t^6}{6}\right|_0^5 = 156.25 \text{ kJ}$$

Alternatively,

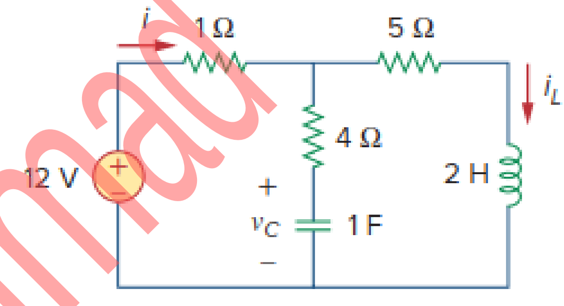
$$w\Big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li^2(0) = \frac{1}{2}(5)(2\times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

Example 6.9

Consider the circuit in Fig.(a). Under dc conditions, find:

a) i, v_C, and i_L,

b) the energy stored in the capacitor and inductor.

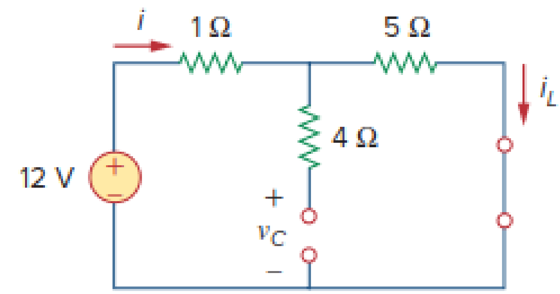


(a)

Solution

a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. (b).

$$i = i_L = \frac{12}{1+5} 2\,\text{A}$$



(b)

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence, $v_C = 5i = 10\,\text{V}$

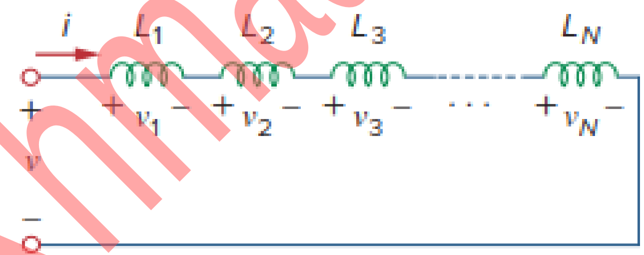
b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10)^2 = 50\,\text{J}; \qquad w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2)^2 = 4\,\text{J}$$

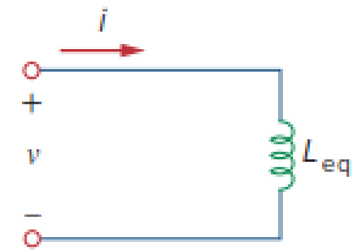
6.4 Series and Parallel Inductors

Series inductors

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

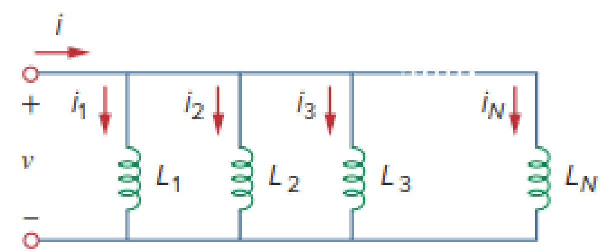


$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$



Parallel inductors

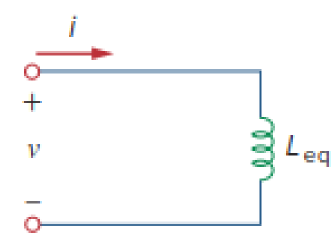
The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_N}$$

For two inductors in parallel ($N = 2$),

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

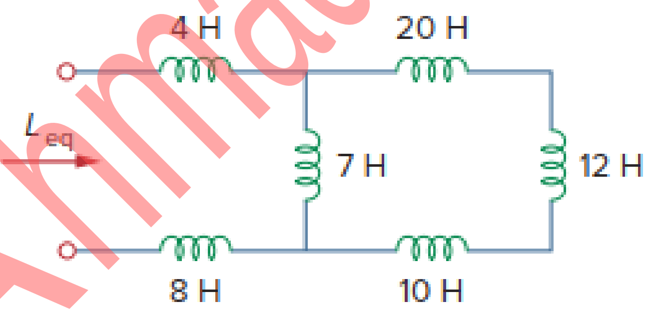


Important characteristics of the basic elements.

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit

Example 6.10

Find the equivalent inductance of the circuit shown in Fig.



Solution

The 10-H, 12-H, and 20-H inductors are in series; thus,

$$10 + 12 + 20 = 42\,\text{H}$$

This 42-H is in parallel with the 7-H :

$$\frac{7 \times 42}{7 + 42} = 6\,\text{H}$$

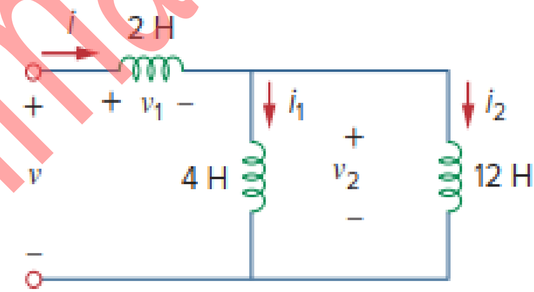
This 6-H in series with 4-H and 8-H. Hence,

$$L_{eq} = 4 + 6 + 8 = 18\,\text{H}$$

Example 6.11

For the circuit in Fig. , $i(t) = 4(2 - e^{-10t})$ mA . If $i_2(0) = -1$ mA, find:

$i_1(0)$; $v(t)$, $v_1(t)$, and $v_2(t)$; $i_1(t)$ and $i_2(t)$.



Solution

a) From $i(t) = 4(2 - e^{-10t})$ mA $\Rightarrow i(0) = 4(2-1) = 4$ mA

Since $i = i_1 + i_2 \Rightarrow i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5$mA

b) The equivalent inductance is $L_{eq} = 2 + 4 || 12 = 5\,H$

Thus,

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} = 200e^{-10t} \text{ mV}$$

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t} = 80e^{-10t} \text{ mV}$$

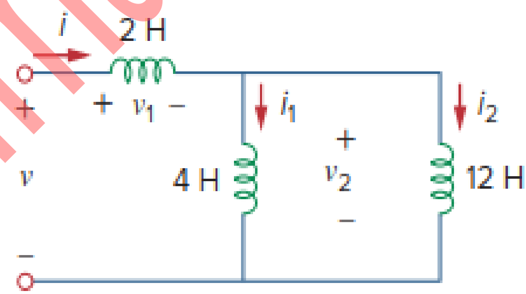
$$v = v_1 + v_2 \Rightarrow v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

c) The current i_1 is obtained as

$$i(t) = 4(2 - e^{-10t}) \text{ mA}$$

$$i_1(t) = \frac{1}{4}\int_0^t v_2 dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t} dt + 5 \text{ mA}$$

$$= 30 \times \left(-\frac{1}{10}\right) e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$



Similarly,

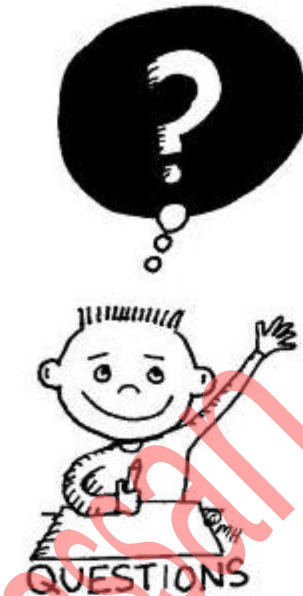
$$i_2(t) = \frac{1}{12}\int_0^t v_2 dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t} dt - 1 \text{ mA}$$

$$= 10 \times \left(-\frac{1}{10}\right) e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

Note that

$$i_1(t) + i_2(t) = i(t)$$

$$8 - 3e^{-10t} \text{ mA} + (-e^{-10t} \text{ mA}) = 4(2 - e^{-10t}) \text{ mA}$$



The end of chapter 6